93

Reihe Ökonomie

Economics Series

Legal Restrictions on Portfolio Holdings:

Some Empirical Results

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Jaroslava Hlouskova, Gabriel S. Lee

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Institut für Höhere Studien (IHS), Wien

Institute for Advanced Studies, Vienna

Contact:

Jaroslava Hlouskova
☎: +43/1/599 91-142
email: hlouskov@ihs.ac.at

Gabriel S. Lee
☎: +43/1/599 91-141
email: gabriel.lee@ihs.ac.at

Abstract

This article investigates the sensitivity analysis of mean-variance portfolio holdings to changes in the upper bounds. The optimization problem studied in this paper is, thus, constrained by a restriction that no more than certain portion of wealth can be invested in any one security. Our empirical results show that for both risk tolerant as well as for risk averse investors, the performance and expected returns of mean-variance efficient portfolios under the legal restrictions are lower and the variance are higher than the corresponding ones without the restriction.

Keywords

Upper bound constraint, portfolio holdings, parametric quadratic programming

JEL Classifications

C61, G11

Comments

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Contents

1. Introduction 1

2. Sensitivity Analysis of Parametric Quadratic Programming: Some Simple Cases 3

3. Some Measures for Trade off 7

4. Some Empirical Results 14

5. Conclusion 21

References 24

1. Introduction

The normative implication drawn from the Mean-Variance (M-V) Capital Asset Pricing Model and the Arbitrage Pricing Theory is that investors should diversify portfolio holdings. Best and Grauer (1991a, 1991b), and Green and Holifield (1992), however, present empirical and theoretical results that, both in theory and in practical implementations, the connection between diversification and mean-variance efficiency is not only tenuous but also problematic: the portfolios constructed using Markowitz (1952) M-V approach involve only a small number of assets or extreme positions on securities. Furthermore, the sampling error in estimates of the weights of an efficient portfolio is often large (Britten-Jones, 1999). Best and Grauer (1985), and Green (1986) also document that positively weighted portfolios are hardly ever observed when M-V portfolios are generated from historical data. Indeed, Eichhorn, Gupta and Stubbs (1998) indicate that, in practice, the composition of portfolio which lies on the efficient frontier does not replicate the mix of risk and return that the Markowitz's (1952) M-V approach predicts.

In response to this apparent contradiction between theory and practice, portfolio managers often implement the M-V approach, willingly or forced, with a set of constraints that enforces diversification.[1] For example, in the U.S. or Europe, investment institutions are often restricted by law that no more than certain percent of the funds of an investment company can be invested in any one security. Additionally, such law also stipulates that the sum of all security holdings that exceed x percent, say five percent, weight of the funds should not exceed certain percent of the total funds. The main purported intention of this law is to enforce portfolio managers to diversify and, in hope, to achieve a balance between being on the efficient frontier of the unrestricted M-V problem and reducing the maximum risk.

By construction, such enforced laws on portfolio holdings achieve a certain level of diversification, but the question is at what costs.[2] The purpose of this paper is to investigate the questions of (i) whether this kind of laws actually leads to a balance between being on the efficient frontier of the unrestricted M-V problem and reducing the maximum risk, and (ii) whether the enforced constraint is too restrictive for various types of investors. In order to shed light on these questions, we propose several measures for the trade-off between reducing the maximum variance and preserving the efficient frontier of the unrestricted M-

[1] A set of such constraints are, for example, no short sales, transaction costs, sector constraints, upper bounds, etc.

[2] Eichhorn, Gupta and Stubbs (1998) show the benefits of implementing restricted portfolio optimization.

V problem when such laws are imposed. Our empirical results based on our proposed measures suggest that the level of diversification enforced by investment laws could be too restrictive for risk tolerant as well as for risk averse investors.

The paper is organized as follows. In Section 2, we briefly introduce the general Parametric Quadratic Programming (PQP) problem and, for an exposition purpose, we also present some simple cases for the sensitivity analysis of M-V portfolio holdings to changes in the upper bounds. In Section 3, we propose various models for measuring the trade-off. Section 4 presents empirical results using Austrian stock market data.[3] Section 5 concludes and summarizes the paper.

2. Sensitivity Analysis of Parametric Quadratic Programming: Some Simple Cases

The M-V portfolio selection problem of Markowitz (1952) and Sharpe (1970) can be formulated as the PQP problem described in Best (1996). The model which we study is similar to that developed in Best and Grauer (1991a), who also use the PQP; hence, expository comments will be brief. General PQP problem is as follows

$$\max_x \left\{ (c+tp)^T x - \frac{1}{2} x^T \Sigma x \mid Ax \leq d + tq \right\}, \tag{1}$$

[3] We use Austrian data since there is an explicit law on investment funds stipulating such upper bound constraint (Austrian Investment Law of 1993).

where c, x and p are n-vectors; Σ is an $n \times n$ symmetric positive semi-definite matrix; A is an $m \times n$ constraint matrix; d, q are m-vectors, t is the PQP parameter, and T denotes transpose.[4]

For exposition and motivation purposes, we illustrate some simple scenarios when an upper bound is imposed on the Markowitz's original M-V portfolio selection problem. The PQP in equation (1) can be rewritten as the following optimization problem

$$\max_x \{-\frac{1}{2}x^T \Sigma x \mid e^T x = 1,\ \mu^T x = \mu_p,\ 0 \leq x \leq be\}, \tag{2}$$

where $c = p = 0_n, A^T = (e, -e, \mu, -\mu, I, -I), d^T = \left(1, -1, \mu_p, -\mu_p, be^T, 0_{2n}^T\right), q^T = \left(0_{n+4}, e^T\right)$, and $t = b$ with μ being an n-vector of expected returns, Σ being an $n \times n$ covariance matrix, 0_n^T being an n-vector of zeros, e being a n-vector of ones, and μ_p varies from μ_l to μ_u which are defined as follows:

$$\begin{aligned} \mu_l &= \min_x \ \{\mu^T x \mid e^T x = 1,\ 0 \leq x \leq be\}, \\ \mu_u &= \max_x \ \{\mu^T x \mid e^T x = 1,\ 0 \leq x \leq be\}, \end{aligned}$$

with b being such that $1/n \leq b \leq 1$. Note that equation 2 has a feasible solution

[4] All non-transposed vectors are column vectors.

only if $nb \geq 1$.[5]

As b is decreased from 1 to $1/n$, the set of feasible M-V combinations (μ_p, σ_p^2) that has the same form in the M-V space as that of the standard portfolio selection problem (i.e., without upper bounds, see Markowitz(1986) for details) will eventually shrink. Figure 2.1 shows the M-V frontiers when no upper bound and an upper bound are imposed.

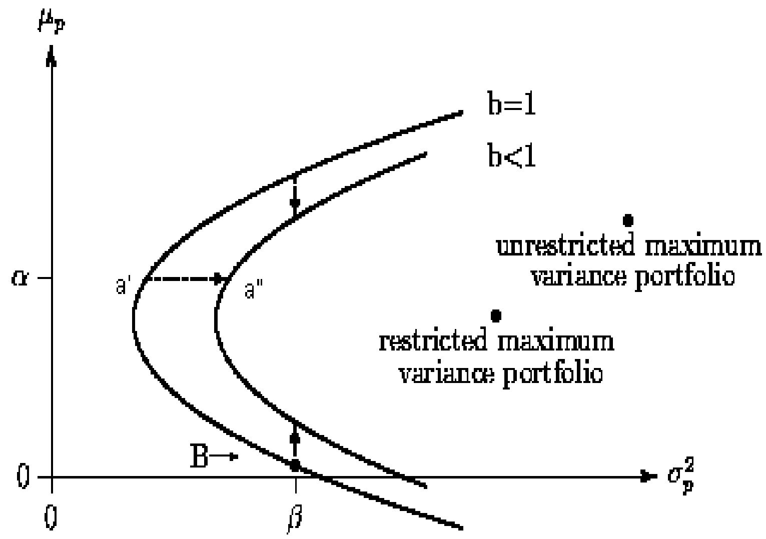

Figure 2.1: Mean-Variance Frontiers

[5]By the standard (or unrestricted) portfolio selection problem, we implicitly take equation (2) without the constraint $x \leq be$.

Figure 2.1, which is artificially generated, leads one to immediately observe the following well-known results. First, the expected return, given variance of return, is lower than or equal to that of the unrestricted problem. Second, the maximum variance is less than or equal to the maximum variance of the standard problem. Third, tracking funds may see their tracking error variance increases (e.g., consider the movement from a' to a'' where $a' < a''$). Lastly, some inefficient parts of the frontier may no longer be feasible.

Figure 2.2 presents a case where the portion of the unrestricted M-V frontier coincides with that of the restricted frontier, whereas Figure 2.3 shows the case where there is no "common segment". Again, we outline some of the well-known results. First, taking the Sharpe ratio as the performance measure, index funds will not be affected if the tangency portfolio is the same for both restricted and unrestricted problems. Second, index funds will have a worse Sharpe ratio if the tangency portfolio is affected.

The qualitative results from these simple situations of the restricted M-V problems motivate and lead us to quantify the actual trade off between being on the efficient frontier of the unrestricted M-V problem and reducing the maximum risk.

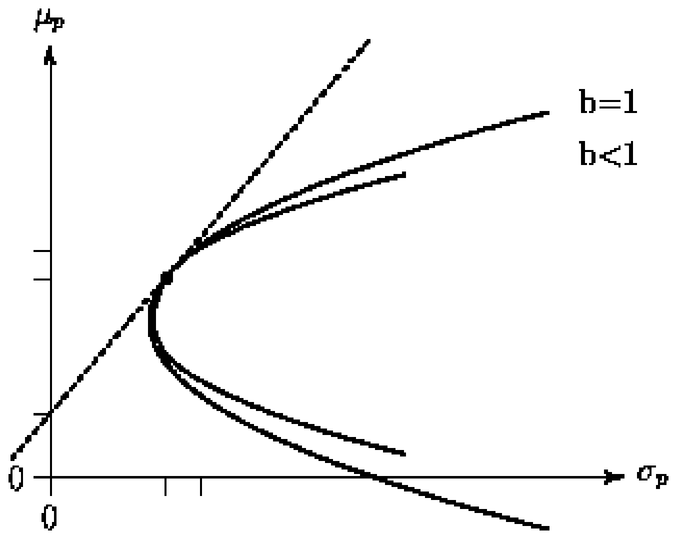

Figure 2.2: Mean-Variance Frontiers with Common Segments

3. Some Measures for Trade off

This sections addresses a question of what is the most "convenient" b, i.e., the upper bound parameter that will achieve the balance between being on the efficient frontier of the standard problem and reducing the maximum risk.[6] To answer this question, we formulate the model that measures the trade off under the various choices for the parameter b.

[6] We assume that the exact distribution of assets' returns is known.

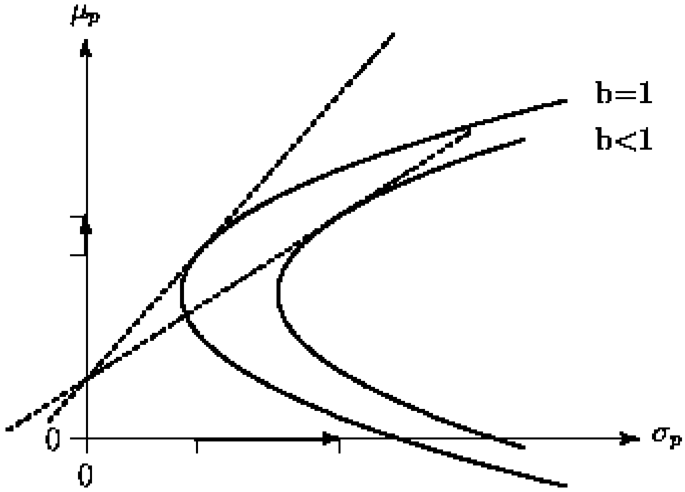

Figure 2.3: Mean-Variance Frontiers without Common Segments

In our formulation of the trade off measure, we re-introduce the original restricted and unrestricted M-V problems. The restricted problem (2) is

$$\max_x \{-\frac{1}{2}x^T\Sigma x \mid e^Tx = 1,\ \mu^Tx = \mu_p,\ 0 \leq x \leq be\}$$

and the unrestricted problem is

$$\max_x \{-\frac{1}{2}x^T\Sigma x \mid e^Tx = 1,\ \mu^Tx = \mu_p,\ x \geq 0\}. \tag{3}$$

Faced with these two M-V optimization problems, we propose a measure that, according to the chosen parameters, reduces the maximum portfolio variance but at the same time tries to preserve the portfolios to lie on the efficient frontier of the standard problem. Before justifying and explaining our measure, the key idea can be stated in the following problem

$$\min_{b} \left\{ H(b) = \frac{c_1 V_{\min}(b) + c_2 G(b) + c_3 P_{\max}(b)}{c_4 V(b) + c_5 P(b)} \mid \frac{1}{n} \leq b < 1 \right\}, \tag{4}$$

where $V_{\min}(b)$, $G(b)$, $P_{\max}(b)$, $V(b)$ and $P(b)$ are defined for $1/n \leq b < 1$ as

$$\begin{aligned} V_{\min}(b) &\equiv \sqrt{(x_5^r)^T \Sigma x_5^r} - \sqrt{(x_5^u)^T \Sigma x_5^u}, \\ G(b) &\equiv \sqrt{(x_2^r)^T \Sigma x_2^r} - \sqrt{(x_3^r)^T \Sigma x_3^r}, \\ P_{\max}(b) &\equiv \mu^T (x_2^u - x_2^r), \\ V(b) &\equiv \sqrt{(x_4^u)^T \Sigma x_4^u} - \sqrt{(x_4^r)^T \Sigma x_4^r}, \\ P(b) &\equiv \mu^T (x_1^r - x_1^u), \end{aligned}$$

and c_i equals zero or one for $i = 1, \ldots, 5$ with c_4 and c_5 not being equal zero at the same time. The quantities $x_1^r, x_1^u, x_2^r, x_2^u, x_3^r, x_4^r, x_4^u, x_5^r$, and x_5^u are as follows (the superscripts r and u denote "restricted" and "unrestricted" respectively):

$$\begin{aligned}
x_1^r &= \text{argmin } \{\mu^T x \mid e^T x = 1,\ 0 \leq x \leq be\}, \\
x_1^u &= \text{argmin } \{\mu^T x \mid e^T x = 1,\ x \geq 0\}, \\
x_2^r &= \text{argmax } \{\mu^T x \mid e^T x = 1,\ 0 \leq x \leq be\}, \\
x_2^u &= \text{argmax } \{\mu^T x \mid e^T x = 1,\ x \geq 0\}, \\
x_3^r &= \text{argmin } \{x^T \Sigma x \mid e^T x = \mu^T x_2^r,\ e^T x = 1,\ x \geq 0\}, \\
x_4^r &= \text{argmax } \{x^T \Sigma x \mid e^T x = 1,\ 0 \leq x \leq be\}, \\
x_4^u &= \text{argmax } \{x^T \Sigma x \mid e^T x = 1,\ x \geq 0\}, \\
x_5^r &= \text{argmin } \{x^T \Sigma x \mid e^T x = 1,\ 0 \leq x \leq be\}, \\
x_5^u &= \text{argmin } \{x^T \Sigma x \mid e^T x = 1,\ x \geq 0\}.
\end{aligned}$$

Figure 3.1 shows the functions $V_{\min}(b)$, $G(b)$, $P_{\max}(b)$, $V(b)$ and $P(b)$ in the Mean-Standard Deviation (M-SD) space and their relationships to the restricted and unrestricted problems.[7]

If one asks for compelling theoretical justifications for our proposed measure, there is none. We accept the fact that our measure is *ad hoc*. Furthermore, we do

[7] One could use other *ad hoc* metrics such as absolute minimum difference, quadratic difference, etc to measure the trade off.

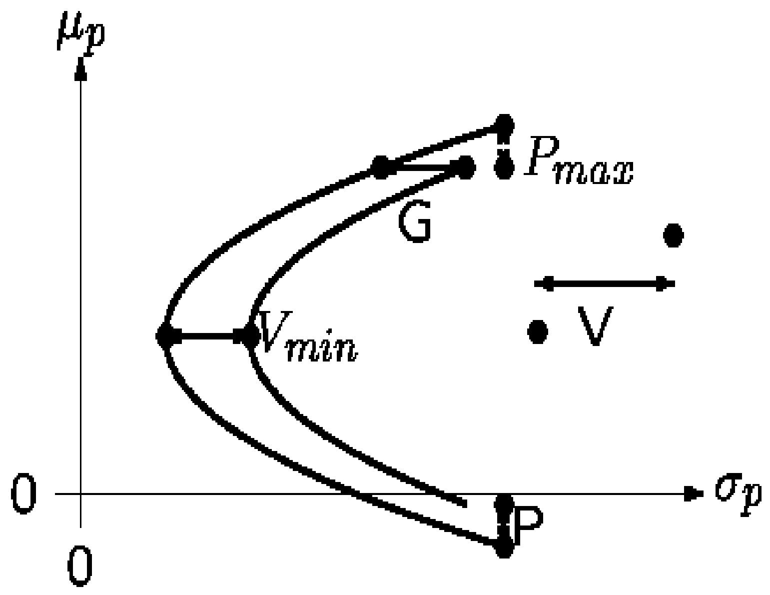

Figure 3.1: Summary of Various Functions on Mean-S.D. Space

not claim that our metric is the most appropriate one. But it seems natural from a policy-making perspective to measure the distance between the two frontiers at various points as well as to measure the reduction in the variance of the maximum variance portfolio.

The details of the measure in equation (4) are based on the following criteria. First, we wish to increase the difference between the maximum standard deviations of the unrestricted and restricted problems; i.e., $V(b)$,[8] and the differ-

[8]Note that the problems defining x_4^u and x_4^r are non-convex problems and in general may have

ence between minimum expected returns of the restricted and unrestricted M-SD portfolio problems; i.e. $P(b)$. In doing so, we decrease the maximum risk and shrink the inefficient part of the M-SD frontier. Second, we wish to minimize the difference between (i) the minimum standard deviations of the portfolios of the restricted and unrestricted problems, $V_{\min}(b)$, (ii) the maximum standard deviation of the efficient portfolio of the restricted problem and the standard deviation of the efficient portfolio of the unrestricted problem with the same expected return as the former portfolio, $G(b)$, and (iii) the maximum expected returns of the unrestricted and restricted M-V portfolio selection problems, $P_{\max}(b)$. By minimizing all $V_{\min}(b), G(b)$ and $P_{\max}(b)$, we reduce the "area" between the efficient frontiers of the restricted and unrestricted portfolio selection problems, and hence we minimize the negative effect of the upper bounds.

A further look at the measure in equation (4) reveals that we wish to make the term in the nominator of $H(b)$, i.e., $c_1 V_{\min}(b) + c_2 G(b) + c_3 P_{\max}(b)$, small but at the same time to make the denominator of $H(b)$, i.e., $c_4 V(b) + c_5 P(b)$, large. One way of resolving these two conflicting goals is to minimize the objective function $H(b)$. Moreover, note that by increasing the upper bound b (i.e., the

many local maxima. However because they are convex maximization problems, their optimum must occur at an extreme point. Because of the simple nature of the constraints, it is easy to enumerate all such extreme points and choose the best one.

constraint becomes less binding), the denominator, $c_4 V(b) + c_5 P(b)$ will decrease. That is, as $c_4 V(b) + c_5 P(b)$ is a decreasing function in b , assuming that the set

$$\{b \mid c_1 V_{\min}(b) + c_2 G(b) + c_3 P_{\max}(b) = 0,\ 1/n \leq b \leq 1\}$$

is non-empty set, our intention is to find a minimum upper bound b for which the function $c_1 V_{\min}(b) + c_2 G(b) + c_3 P_{\max}(b)$ equals zero. In the process of such minimization, the maximization of the function $c_4 V(b) + c_5 P(b)$ for all values b such that the function $H(b)$ reaches zero, is achieved. This is given by:

$$\min \{b \mid c_1 V_{\min}(b) + c_2 G(b) + c_3 P_{\max}(b) = 0,\ 1/n \leq b \leq 1\}, \tag{5}$$

where also the minimization objective of $H(b)$ is captured. The equality $c_1 V_{\min}(b) + c_2 G(b) + c_3 P_{\max}(b) = 0$ insures that some part of the efficient frontier of the unrestricted problem is not violated; i.e., some investors will not be worse off when the upper bound is imposed.[9] Further, the fact that we are minimizing the upper bound b subject to the constraints described in (5) ensures that the denominator in the objective function of problem (4) $c_4 V(b) + c_5 P(b)$ is maximized.

[9] For example, if $c_1 = 1$ and $c_2 = c_3 = 0$ then the part of the efficient frontier of the unrestricted problem corresponding to the risk averse investors is not violated.

4. Some Empirical Results

As there is an explicit investment law on the upper bounds of portfolio holdings in Austria, we use the group of 30 most liquid Austrian stocks to analyze the models in equations (4) and (5).[10] We use the daily returns of 30 continuously traded stocks, which are listed on the Austrian Stock Exchange and ranked according to the turnover. We focus on two time periods both containing 285 data points: the first period contains the data from April 10, 1995 to June 7, 1996 and the second period contains the data from June 10, 1996 to August 1, 1997. The stocks[11] are identical for both periods.

In order to obtain robust results, we analyze various combinations of the models in (4) and (5) according to the degree of investor's risk aversion, which is determined by the coefficients c_1, c_2, and c_3. We classify the models into three classes. The first class represents the most risk averse investors; i.e. $c_1 = 1$ and $c_2 = c_3 = 0$. The second class represents the investors who are less risk averse; i.e., $c_1 = 0$ or 1, $c_2 = 1$, and $c_3 = 0$. The risk tolerant investors are classified in

[10] The portfolio optimization was performed with both GAUSS and software provided by Financiometrics, Inc., Orinda, California. The authors gratefully acknowledges the assistance of Financiometrics.

[11] Data of daily returns were obtained from "Österreichische Kontrolbank". The elements of the vector of expected returns μ and the covariance matrix Σ were estimated by historical means, variances and covariances. These estimations of the parameters were assumed to result in the "true" values of expected returns, variances and covariances.

Table 4.1: The Summary of the Models' Results

H_i	b_{opt} for 1st period	b_{opt} for 2nd period
$H_1 = \frac{V_{min}}{V}$	0.5	0.4
$H_2 = \frac{V_{min}}{V+P}$	0.5	0.4
$H_3 = \frac{V_{min}+G}{V}$	0.8	0.5
$H_4 = \frac{V_{min}+G}{V+P}$	0.8	0.5
$H_5 = \frac{G}{V}$	0.8	0.5
$H_6 = \frac{G}{V+P}$	0.8	0.5
$H_7 = \frac{P_{max}}{V}$	0.9	0.9
$H_8 = \frac{P_{max}}{V+P}$	0.9	0.9
$H_9 = \frac{V_{min}+P_{max}}{V}$	0.9	0.9

the last group with $c_1 = 0$ or 1, $c_2 = 0$ or 1, and $c_3 = 1$. Table 4.1 shows the results of the numerical minimization procedures of models for both periods.

The first set of minimizations involves the "most risk averse" investors. The measures $H_1(b)$ and $H_2(b)$ deal with the function $V_{\min}(b)$, which is the difference between the minimum standard deviation of the restricted and unrestricted problems. Thus, we classify the measures $H_1(b)$ and $H_2(b)$ to represent the class of investors whose objective is solely to obtain the lowest possible risk on the efficient frontier. The measure $H_1(b)$ is normalized by $V(b)$ (the difference between the maximum standard deviations of the unrestricted and restricted problems) and $H_2(b)$ is normalized by the function $V(b) + P(b)$.[12] For both $H_1(b)$ and $H_2(b)$, their minimums are reached for $b = 0.5$ in the first and $b = 0.4$ in the second

[12]All the functions H_i, $i = 1, \ldots, 9$ are normalized by either V or $V + P$.

period, respectively. The next set of minimizations deals with measures which could represent the "average risk averse" investors. All measures in the second class, $H_i(b)$, $i = 3, \ldots, 6$, incorporate the function $G(b)$ and do not incorporate the function $P_{\max}(b)$. Thus, all $H_i(b)$, $i = 3, \ldots, 6$ present some degree of risk tolerance through the function $G(b)$. Throughout the second set of minimization procedures, the minimums are reached for $b = 0.8$ in the first and $b = 0.5$ in the second period, respectively. The last set of minimizations involves the function, $P_{\max}(b)$, which is the difference between the maximum expected returns of efficient portfolios of the unrestricted and restricted problems. Thus, the function $P_{\max}(b)$, intends to capture efficient portfolio of both problems that reflects the most risk tolerant investors. Consequently, the last class of models represented by objective functions $H_i(b)$ for $i = 7, 8, 9$, represent with the most risk tolerant investors. The minimization results for $H_i(b)$, $i = 7, 8, 9$ indicate that the minimums are reached for $b = 0.9$ both in the first and second periods.[13]

Several points from the minimizations are worth mentioning. First, for all classes of investors, our proposed measures are not in line with the Austrian ten percent upper bound rule ($b = 0.1$). Even for the measures that reflect the most

[13]As our objective is to find a numerical solution for the upper bound, b, that would minimize the objective equation (4), our approach does not place a set of statistical tests on our measure. Consequently, our approach differs from Britten-Jones (1999) who tests a limit on portfolio holding using statistical methods and Green and Hollifield (1992) who test restrictions due to measurement errors.

risk averse investors, the minimum upper bound is at forty percent ($b = 0.4$) (for the second period). Second, as expected, the upper bounds increase with the investors' risk tolerance: ninety percent ($b = 0.9$) for the risk tolerant investors and forty percent ($b = 0.4$) for the models capturing risk averse investors. Lastly, the function $P(b)$, which is the difference between the minimum expected returns of the restricted and unrestricted M-V portfolio problems, does not seem to have any effect on the results of minimization procedures. The minimization results are robust with respect to the normalization through $V(b) + P(b)$ or $V(b)$ alone. In other words, the minimizations seem not to be affected by the portfolio holdings that lie on the inefficient part of the M-SD frontier.

We use function $H_6(b)$ to obtain a better insight into at the numerical minimization of (4) and (5).[14] The first column of Tables 4.2 and 4.3 presents the monotonically increasing values of upper bound, b. The values of functions $G(b), P(b), V(b)$ and $H_6(b)$ are calculated for the corresponding values of the upper bound b. The last column in both tables presents a range of expected returns where efficient frontiers of both restricted and unrestricted problems coincide.

As indicated previously in Table 4.1, both Tables 4.2 and 4.3 show that the

[14]We could have used any of $H_i(b)$ as all measures give the same qualitative results. But as $H_6(b)$ represents an average investor who faces some degrees of both risks and riskless investment, and it highlights the differences across the two subperiods, we choose $H_6(b)$ for further empirical explanations.

Table 4.2: First Period Results for H6

b	G	P	V	H_6=G/(V+P)	Expected Returns
0.033	0.258	0.238	2.571	0.092	$\emptyset$
0.1	0.298	0.118	2.077	0.087	$\emptyset$
0.2	0.134	0.077	1.842	0.070	$\emptyset$
0.3	0.143	0.063	1.576	0.087	[-0.053,-0.003] $\cup$ [0.271,0.321]
0.4	0.152	0.053	1.342	0.109	[-0.064,0.035] $\cup$ [0.210,0.385]
0.5	0.185	0.042	1.071	0.165	[-0.074,0.450]
0.6	0.057	0.034	0.922	0.059	[-0.083,0.517]
0.7	0.002	0.025	0.728	0.003	[-0.091,0.583]
0.8	0.	0.017	0.506	0.	[-0.100,0.614]
0.9	0.	0.008	0.261	0.	[-0.133,1.901]

Table 4.3: Second Period Results for H6

b	G	P	V	H_6=G/(V+P)	Expected Returns
0.033	0.202	0.243	2.077	0.087	$\emptyset$
0.1	0.115	0.137	1.678	0.063	$\emptyset$
0.2	0.106	0.093	1.433	0.069	[0.194,0.219]
0.3	0.174	0.065	1.213	0.136	[-0.028,-0.008] $\cup$ [0.117,0.272]
0.4	0.089	0.051	1.012	0.084	[-0.052,0.298]
0.5	0.	0.042	0.774	0.	[-0.071,0.335]
0.6	0.	0.033	0.676	0.	[-0.095,0.336]
0.7	0.	0.025	0.544	0.	[-0.113,0.337]
0.8	0.	0.016	0.384	0.	[-0.132,0.338]
0.9	0.	0.008	0.201	0.	[-0.140,0.339]

minimum value of the objective function H_6 is reached for $b \in \{0.8, 0.9\}$ in the first period and $b \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ in the second period, respectively. The corresponding expected returns of b fall in the range of expected returns where efficient frontier of restricted and unrestricted problems coincide. For comparison purpose, for $b = 0.1$ (the upper bound stated by the Austrian Law) the objective function H_6 obtains the value of 0.087 for the first period and 0.063 for the second period, and in both periods there is no interval of expected returns where efficient frontiers of both restricted and unrestricted problems coincide. The existence of a "common" segment is an important information that indicates the degree of restrictiveness of upper bounds.[15]

Figures 4.1, 4.2, 4.3 and 4.4,[16] which plot the M-SD frontiers of both restricted and unrestricted problems with $b = 0.1$ for both periods show that there is no common segment, and thus the ten percent constraint seems to be a quite restrictive constraint - less restrictive for the second period than for the first one. Mutual funds which are active during these two periods and track a certain value of risk or expected return or maximize their performance measured by the Sharpe

[15]Using the tool of the PQP, one can find the interval for a risk tolerance parameter τ which describes the segment on the efficient frontier of the unrestricted problem which coincides with a part of the efficient frontier of the restricted problem, or determine that such a segment does not exist. A similar but less complex problem is solved by Best and Grauer (1991b).

[16]Figures 6 and 8 are output files from the GAUSS program and show the same frontiers as Figures 5 and 7 but with finer scaling.

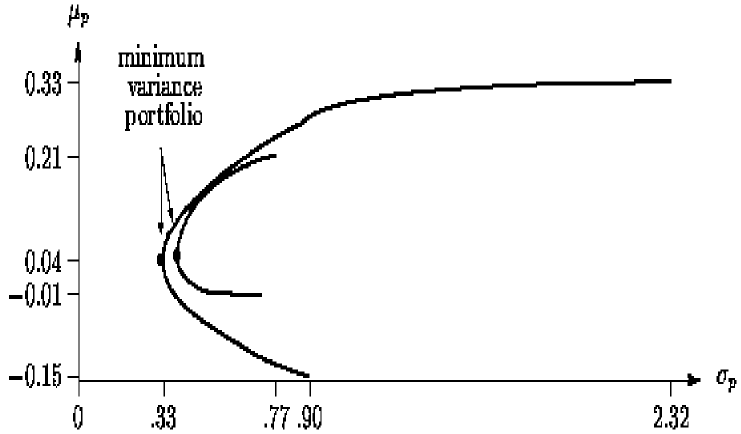

Figure 4.1: M-V Frontiers for the First Period, b=0.1

ratio would be worse off when the ten percent restriction is imposed. Note that for the case of the second period (see Figures 4.3 and 4.4) the risk averse investors who tend to minimize the variance of returns of their portfolios are more negatively affected (or affected in more negative terms) by the ten percent constraint then the less risk averse investors. For standard deviation in the range of [0.5,0.6] and the expected returns in the interval [0.14,0.18], the efficient frontiers nearly coincide in the second period. Consequently, the ten percent constraint is less restrictive for investors who track the standard deviation and the expected return

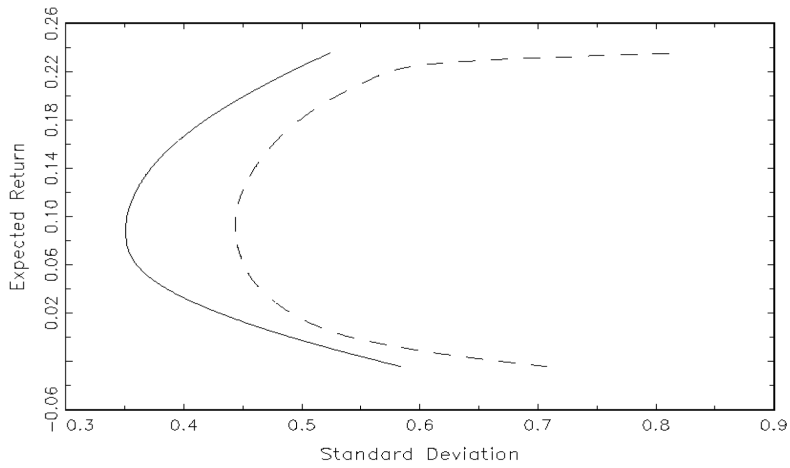

Figure 4.2: M-V Frontiers for the First Period, b=0.1 (Scaled Version)

in the ranges specified above.

5. Conclusion

We use the framework of the PQP to examine the empirical aspects of the M-V problems and sensitivity analysis. Using Austrian stock market data, we investigate the questions of (i) whether upper bounds on portfolio holdings lead to a "balance" between being on the efficient frontier of the unrestricted M-V problem and reducing the maximum risk, and (ii) whether the enforced constraint is

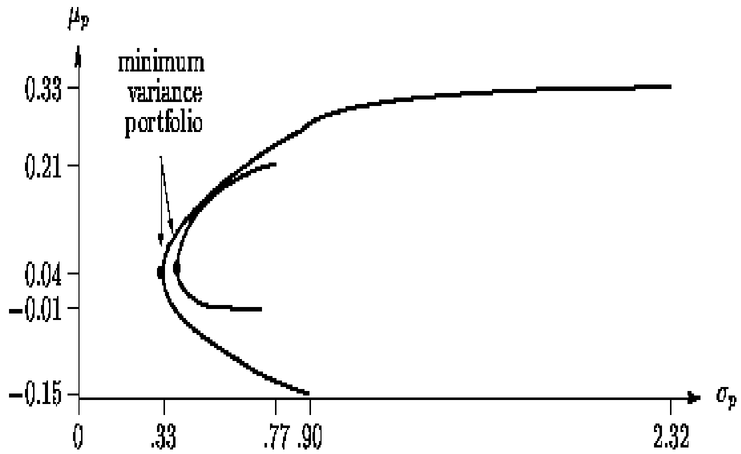

Figure 4.3: M-V Frontiers for the Second Period, b=0.1

restrictive for investors. To shed light on these questions, we propose several measures for the trade-off between being on the efficient frontier of the unrestricted M-V problem and reducing the maximum variance under the imposed constraint.

Our empirical results suggest that the level of diversification enforced by investment laws can be quite restrictive for both risk averse and risk tolerant investors. Our proposed measures, given sample distribution, applied on the data set indicate that at least forty percent rule is needed to achieve the balance between being on the efficient frontier of the unrestricted M-V problem and reducing

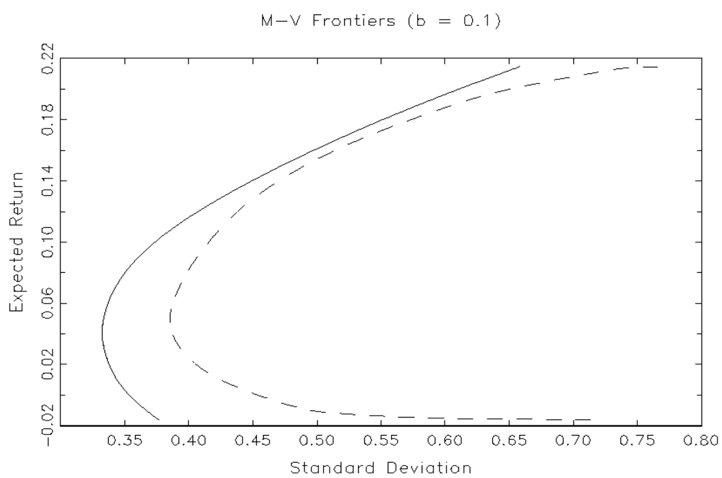

Figure 4.4: M-V Frontier for the Second Period, b=0.1 (Scaled Version)

the maximum variance. This empirical finding indicate that the current European (ten percent) and the U.S. (five percent) restrictions on portfolio holdings might be too restrictive for the trade off that we discussed in this paper.

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